

Introduction to Information Retrieval

<http://informationretrieval.org>

IIR 18: Latent Semantic Indexing

Wiltrud Kessler & Hinrich Schütze

Institute for Natural Language Processing, Universität Stuttgart

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Overview

- 1 Recap
- 2 Latent semantic indexing
- 3 Dimensionality reduction
- 4 LSI in information retrieval
- 5 Clustering

Outline

- 1 Recap
- 2 Latent semantic indexing
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- 5 Clustering

Applications of clustering in IR

Application	What is clustered?	Benefit	Example
Search result clustering	search results	more effective information presentation to user	
Scatter-Gather	(subsets of) collection	alternative user interface: "search without typing"	
Collection clustering	collection	effective information presentation for exploratory browsing	McKeown et al. 2002, news.google.com
Cluster-based retrieval	collection	higher efficiency: faster search	Salton 1971

K-means algorithm

```
K-MEANS( $\{\vec{x}_1, \dots, \vec{x}_N\}, K$ )
1   $(\vec{s}_1, \vec{s}_2, \dots, \vec{s}_K) \leftarrow \text{SELECTRANDOMSEEDS}(\{\vec{x}_1, \dots, \vec{x}_N\}, K)$ 
2  for  $k \leftarrow 1$  to  $K$ 
3  do  $\vec{\mu}_k \leftarrow \vec{s}_k$ 
4  while stopping criterion has not been met
5  do for  $k \leftarrow 1$  to  $K$ 
6      do  $\omega_k \leftarrow \{\}$ 
7      for  $n \leftarrow 1$  to  $N$ 
8          do  $j \leftarrow \arg \min_{j'} |\vec{\mu}_{j'} - \vec{x}_n|$ 
9               $\omega_j \leftarrow \omega_j \cup \{\vec{x}_n\}$  (reassignment of vectors)
10     for  $k \leftarrow 1$  to  $K$ 
11         do  $\vec{\mu}_k \leftarrow \frac{1}{|\omega_k|} \sum_{\vec{x} \in \omega_k} \vec{x}$  (recomputation of centroids)
12 return  $\{\vec{\mu}_1, \dots, \vec{\mu}_K\}$ 
```

Initialization of K -means

- Random seed selection is just one of many ways K -means can be initialized.
- Random seed selection is not very robust: It's easy to get a suboptimal clustering.
- Better heuristics:
 - Select seeds not randomly, but using some heuristic (e.g., filter out outliers or find a set of seeds that has “good coverage” of the document space)
 - Use hierarchical clustering to find good seeds (next class)
 - Select i (e.g., $i = 10$) different sets of seeds, do a K -means clustering for each, select the clustering with lowest RSS

Take-away today

- Latent Semantic Indexing (LSI) / Singular Value Decomposition: The math
- SVD used for dimensionality reduction
- LSI: SVD in information retrieval
- LSI as clustering

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Recall: Term-document matrix

	Anthony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
anthony	5.25	3.18	0.0	0.0	0.0	0.35
brutus	1.21	6.10	0.0	1.0	0.0	0.0
caesar	8.59	2.54	0.0	1.51	0.25	0.0
calpurnia	0.0	1.54	0.0	0.0	0.0	0.0
cleopatra	2.85	0.0	0.0	0.0	0.0	0.0
mercy	1.51	0.0	1.90	0.12	5.25	0.88
worser	1.37	0.0	0.11	4.15	0.25	1.95

...

This matrix is the basis for computing [the similarity between](#)

[documents and queries](#). Today: Can we transform this matrix, so

that we get a [better measure of similarity](#) between documents and queries? □

Latent semantic indexing: Overview

- We will **decompose** the term-document matrix into a product of matrices.
- The particular decomposition we'll use: **singular value decomposition** (SVD).
- SVD: $C = U\Sigma V^T$ (where C = term-document matrix)
- We will then use the SVD to compute a **new, improved term-document matrix** C' .
- We'll get **better similarity** values out of C' (compared to C).
- Using SVD for this purpose is called **latent semantic indexing** or LSI. □

Example of $C = U\Sigma V^T$: The matrix C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

This is a standard

term-document matrix. Actually, we use a non-weighted matrix here to simplify the example. □

Example of $C = U\Sigma V^T$: The matrix U

U	1	2	3	4	5
ship	-0.44	-0.30	0.57	0.58	0.25
boat	-0.13	-0.33	-0.59	0.00	0.73
ocean	-0.48	-0.51	-0.37	0.00	-0.61
wood	-0.70	0.35	0.15	-0.58	0.16
tree	-0.26	0.65	-0.41	0.58	-0.09

One row per

term, one column per $\min(M, N)$ where M is the number of terms and N is the number of documents. This is an **orthonormal matrix**:

(i) Row vectors have unit length. (ii) Any two distinct row vectors are orthogonal to each other. Think of the dimensions as

“semantic” dimensions that capture distinct topics like politics, sports, economics. 2 = land/water Each number u_{ij} in the matrix indicates how strongly related term i is to the topic represented by semantic dimension j . □

Example of $C = U\Sigma V^T$: The matrix Σ

Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

This is a square, diagonal

matrix of dimensionality $\min(M, N) \times \min(M, N)$. The diagonal consists of the singular values of C . The magnitude of the singular value measures the importance of the corresponding semantic dimension. We'll make use of this by omitting unimportant dimensions. □

Example of $C = U\Sigma V^T$: The matrix V^T

V^T	d_1	d_2	d_3	d_4	d_5	d_6	
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12	One
2	-0.29	-0.53	-0.19	0.63	0.22	0.41	
3	0.28	-0.75	0.45	-0.20	0.12	-0.33	
4	0.00	0.00	0.58	0.00	-0.58	0.58	
5	-0.53	0.29	0.63	0.19	0.41	-0.22	

column per document, one row per $\min(M, N)$ where M is the number of terms and N is the number of documents. Again: This is an **orthonormal matrix**: (i) Column vectors have unit length. (ii) Any two distinct column vectors are orthogonal to each other. These are again the semantic dimensions from matrices U and Σ that capture distinct topics like politics, sports, economics. Each number v_{ij} in the matrix indicates how strongly related document i is to the topic represented by semantic dimension j . \square

Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	-0.44	-0.30	0.57	0.58	0.25
boat	-0.13	-0.33	-0.59	0.00	0.73
ocean	-0.48	-0.51	-0.37	0.00	-0.61
wood	-0.70	0.35	0.15	-0.58	0.16
tree	-0.26	0.65	-0.41	0.58	-0.09

×

Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22

LSI is

decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.



LSI: Summary

- We've decomposed the term-document matrix C into a product of three matrices: $U\Sigma V^T$.
- The term matrix U – consists of one (row) vector for each term
- The document matrix V^T – consists of one (column) vector for each document
- The singular value matrix Σ – diagonal matrix with singular values, reflecting importance of each dimension
- Next: Why are we doing this? □

Exercise

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22

Verify

that the first document has unit length. Verify that the first two documents are orthogonal.

$$0.75^2 + 0.29^2 + 0.28^2 + 0.00^2 + 0.53^2 = 1.0059$$
$$-0.75 * -0.28 +$$
$$-0.29 * -0.53 + 0.28 * -0.75 + 0.00 * 0.00 + -0.53 * 0.29 = 0$$

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How we use the SVD in LSI

- Key property: Each singular value tells us how important its dimension is.
- By setting less important dimensions to zero, we keep the important information, but get rid of the “details”.
- These details may
 - be **noise** – in that case, reduced LSI is a better representation because it is less noisy.
 - **make things dissimilar that should be similar** – again, the reduced LSI representation is a better representation because it represents similarity better.
- Analogy for “fewer details is better”
 - Image of a blue flower
 - Image of a yellow flower
 - Omitting color makes it easier to see the similarity □

Reducing the dimensionality to 2

U	1	2	3	4	5	
ship	-0.44	-0.30	0.00	0.00	0.00	
boat	-0.13	-0.33	0.00	0.00	0.00	
ocean	-0.48	-0.51	0.00	0.00	0.00	
wood	-0.70	0.35	0.00	0.00	0.00	
tree	-0.26	0.65	0.00	0.00	0.00	
Σ_2	1	2	3	4	5	
1	2.16	0.00	0.00	0.00	0.00	
2	0.00	1.59	0.00	0.00	0.00	
3	0.00	0.00	0.00	0.00	0.00	
4	0.00	0.00	0.00	0.00	0.00	
5	0.00	0.00	0.00	0.00	0.00	
V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00

Actually, we only zero out singular values in Σ . This has the effect of setting the corresponding dimensions in U and V^T to zero when computing the product $C = U\Sigma V^T$. \square

Reducing the dimensionality to 2

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	-0.08
boat	0.36	0.36	0.16	-0.20	-0.02	-0.18
ocean	1.01	0.72	0.36	-0.04	0.16	-0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	-0.39	-0.08	0.90	0.41	0.49
U	1	2	3	4	5	
ship	-0.44	-0.30	0.57	0.58	0.25	
boat	-0.13	-0.33	-0.59	0.00	0.73	
ocean	-0.48	-0.51	-0.37	0.00	-0.61	×
wood	-0.70	0.35	0.15	-0.58	0.16	
tree	-0.26	0.65	-0.41	0.58	-0.09	
Σ_2	1	2	3	4	5	
1	2.16	0.00	0.00	0.00	0.00	
2	0.00	1.59	0.00	0.00	0.00	
3	0.00	0.00	0.00	0.00	0.00	×
4	0.00	0.00	0.00	0.00	0.00	
5	0.00	0.00	0.00	0.00	0.00	
V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22



Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	-0.44	-0.30	0.57	0.58	0.25
boat	-0.13	-0.33	-0.59	0.00	0.73
ocean	-0.48	-0.51	-0.37	0.00	-0.61
wood	-0.70	0.35	0.15	-0.58	0.16
tree	-0.26	0.65	-0.41	0.58	-0.09

×

Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22

LSI is

decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.



Original matrix C vs. reduced $C_2 = U\Sigma_2V^T$

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1
C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	-0.08
boat	0.36	0.36	0.16	-0.20	-0.02	-0.18
ocean	1.01	0.72	0.36	-0.04	0.16	-0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	-0.39	-0.08	0.90	0.41	0.49

We can view C_2 as a **two-dimensional** representation of the matrix C . We have performed a **dimensionality reduction** to two dimensions. □

Exercise

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	-0.08
boat	0.36	0.36	0.16	-0.20	-0.02	-0.18
ocean	1.01	0.72	0.36	-0.04	0.16	-0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	-0.39	-0.08	0.90	0.41	0.49

Compute the similarity between d_2 and d_3 for the original matrix and for the reduced matrix.

Why the reduced matrix C_2 is better than C

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1
C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.85	0.52	0.28	0.13	0.21	-0.08
boat	0.36	0.36	0.16	-0.20	-0.02	-0.18
ocean	1.01	0.72	0.36	-0.04	0.16	-0.21
wood	0.97	0.12	0.20	1.03	0.62	0.41
tree	0.12	-0.39	-0.08	0.90	0.41	0.49

Similarity of d_2 and d_3 in the original space: 0. Similarity of d_2 and d_3 in the reduced space: $0.52 * 0.28 + 0.36 * 0.16 + 0.72 * 0.36 + 0.12 * 0.20 + -0.39 * -0.08 \approx 0.52$

“boat” and “ship” are semantically similar. The “reduced” similarity measure reflects this. What

Exercise: Compute matrix product

C_2	d_1	d_2	d_3	d_4	d_5	d_6
ship	0.09	0.16	0.06	-0.19	-0.07	-0.12
boat	0.10	0.17	0.06	-0.21	-0.07	-0.14
ocean	0.15	0.27	0.10	-0.32	-0.11	-0.21
wood	-0.10	-0.19	-0.07	0.22	0.08	0.14
tree	-0.19	-0.34	-0.12	0.41	0.14	0.27
U	1	2	3	4	5	
ship	-0.44	-0.30	0.57	0.58	0.25	
boat	-0.13	-0.33	-0.59	0.00	0.73	
ocean	-0.48	-0.51	-0.37	0.00	-0.61	×
wood	-0.70	0.35	0.15	-0.58	0.16	
tree	-0.26	0.65	-0.41	0.58	-0.09	
Σ_2	1	2	3	4	5	
1	0.00	0.00	0.00	0.00	0.00	
2	0.00	1.59	0.00	0.00	0.00	
3	0.00	0.00	0.00	0.00	0.00	×
4	0.00	0.00	0.00	0.00	0.00	
5	0.00	0.00	0.00	0.00	0.00	
V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22

???????=?

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Why we use LSI in information retrieval

- LSI takes documents that are semantically similar (= talk about the same topics), ...
- ...but are not similar in the vector space (because they use different words) ...
- ...and re-represents them in a reduced vector space ...
- ...in which they have higher similarity.
- Thus, LSI addresses the problems of **synonymy** and **semantic relatedness**.
- Standard vector space: Synonyms contribute nothing to document similarity.
- Desired effect of LSI: Synonyms contribute strongly to document similarity.



How LSI addresses synonymy and semantic relatedness

- The dimensionality reduction forces us to omit a lot of “detail”.
- We have to map different words (= different dimensions of the full space) to the same dimension in the reduced space.
- The “cost” of mapping synonyms to the same dimension is much less than the cost of collapsing unrelated words.
- SVD selects the “least costly” mapping (see below).
- Thus, it will map synonyms to the same dimension.
- But it will avoid doing that for unrelated words. □

LSI: Comparison to other approaches

- Recap: **Relevance feedback** and **query expansion** are used to **increase recall** in information retrieval – if query and documents have no terms in common.
 - (or, more commonly, too few terms in common for a high similarity score)
- LSI **increases recall and hurts precision**.
- Thus, it addresses the same problems as (pseudo) relevance feedback and query expansion . . .
- . . . and it has the same problems. □

Implementation

- Compute SVD of term-document matrix
- Reduce the space and compute reduced document representations
- Map the query into the reduced space $\vec{q}_k = \Sigma_k^{-1} U_k^T \vec{q}$.
- This follows from: $C_k = U \Sigma_k V^T \Rightarrow \Sigma_k^{-1} U^T C = V_k^T$
- Compute similarity of q_k with all reduced documents in V_k .
- Output ranked list of documents as usual
- Exercise: What is the fundamental problem with this approach?



Optimality

- SVD is **optimal** in the following sense.
- Keeping the k largest singular values and setting all others to zero gives you the optimal approximation of the original matrix C . **Eckart-Young theorem**
- Optimal: no other matrix of the same rank (= with the same underlying dimensionality) approximates C better.
- Measure of approximation is Frobenius norm:
$$\|C\|_F = \sqrt{\sum_i \sum_j c_{ij}^2}$$
- So LSI uses the “best possible” matrix.
- There is only one best possible matrix – unique solution (modulo signs).
- Caveat: There is only a tenuous relationship between the Frobenius norm and cosine similarity between documents. □

Data for graphical illustration of LSI

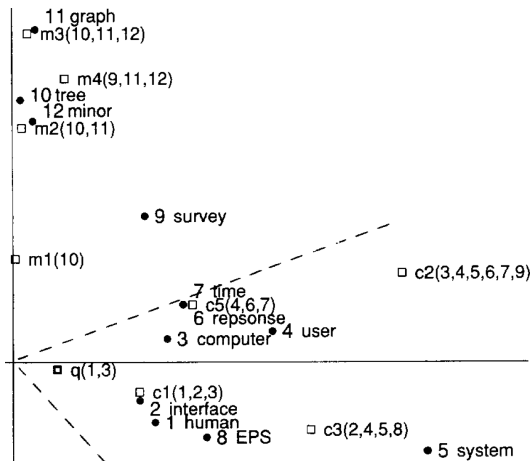
- c_1 Human machine interface for lab abc computer applications
- c_2 A survey of user opinion of computer system response time
- c_3 The EPS user interface management system
- c_4 System and human system engineering testing of EPS
- c_5 Relation of user perceived response time to error measurement
- m_1 The generation of random binary unordered trees
- m_2 The intersection graph of paths in trees
- m_3 Graph minors IV Widths of trees and well quasi ordering
- m_4 Graph minors A survey

The matrix C

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1



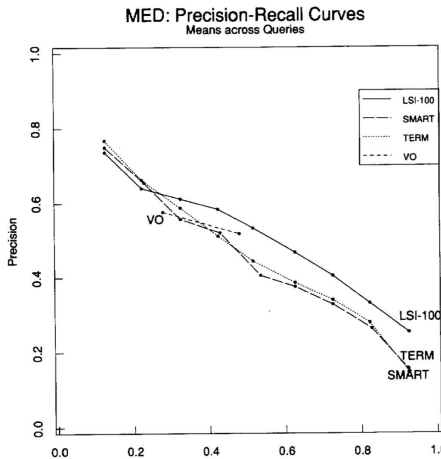
Graphical illustration of LSI: Plot of C_2



2-dimensional plot of C_2 (scaled dimensions). Circles = terms. Open squares = documents (component terms in parentheses). q = query "human computer interaction".

The dotted cone represents the region whose points are within a cosine of .9 from q . All documents about human-computer documents (c1-c5) are near q , even c3/c5 although they share no terms. None of the graph theory documents (m1-m4) are near q . □

LSI performs better than vector space on MED collection



LSI-100 = LSI reduced to

100 dimensions; SMART = SMART implementation of vector space model



Outline

- 1 Recap
- 2 Latent semantic indexing
- 3 Dimensionality reduction
- 4 LSI in information retrieval
- 5 Clustering

Example of $C = U\Sigma V^T$: All four matrices

C	d_1	d_2	d_3	d_4	d_5	d_6
ship	1	0	1	0	0	0
boat	0	1	0	0	0	0
ocean	1	1	0	0	0	0
wood	1	0	0	1	1	0
tree	0	0	0	1	0	1

=

U	1	2	3	4	5
ship	-0.44	-0.30	0.57	0.58	0.25
boat	-0.13	-0.33	-0.59	0.00	0.73
ocean	-0.48	-0.51	-0.37	0.00	-0.61
wood	-0.70	0.35	0.15	-0.58	0.16
tree	-0.26	0.65	-0.41	0.58	-0.09

×

Σ	1	2	3	4	5
1	2.16	0.00	0.00	0.00	0.00
2	0.00	1.59	0.00	0.00	0.00
3	0.00	0.00	1.28	0.00	0.00
4	0.00	0.00	0.00	1.00	0.00
5	0.00	0.00	0.00	0.00	0.39

×

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	-0.75	-0.28	-0.20	-0.45	-0.33	-0.12
2	-0.29	-0.53	-0.19	0.63	0.22	0.41
3	0.28	-0.75	0.45	-0.20	0.12	-0.33
4	0.00	0.00	0.58	0.00	-0.58	0.58
5	-0.53	0.29	0.63	0.19	0.41	-0.22

LSI is

decomposition of C into a representation of the terms, a representation of the documents and a representation of the importance of the "semantic" dimensions.



Why LSI can be viewed as soft clustering

- Each of the k dimensions of the reduced space is one cluster.
- If the value of the LSI representation of document d on dimension k is x , then x is the soft membership of d in topic k .
- This soft membership can be positive or negative.
- Example: Dimension 2 in our SVD decomposition
- This dimension/cluster corresponds to the water/earth dichotomy.
- “ship”, “boat”, “ocean” have negative values.
- “wood”, “tree” have positive values.
- d_1, d_2, d_3 have negative values (most of their terms are water terms).
- d_4, d_5, d_6 have positive values (all of their terms are earth terms).



Take-away today

- Latent Semantic Indexing (LSI) / Singular Value Decomposition: The math
- SVD used for dimensionality reduction
- LSI: SVD in information retrieval
- LSI as clustering

Resources

- Chapter 18 of IIR
- Resources at <http://ifnlp.org/ir>
 - Original paper on latent semantic indexing by Deerwester et al.
 - Paper on probabilistic LSI by Thomas Hofmann
 - Word space: LSI for words