Introduction to Information Retrieval
http://informationretrieval.org

IIR 14: Vector Space Classification

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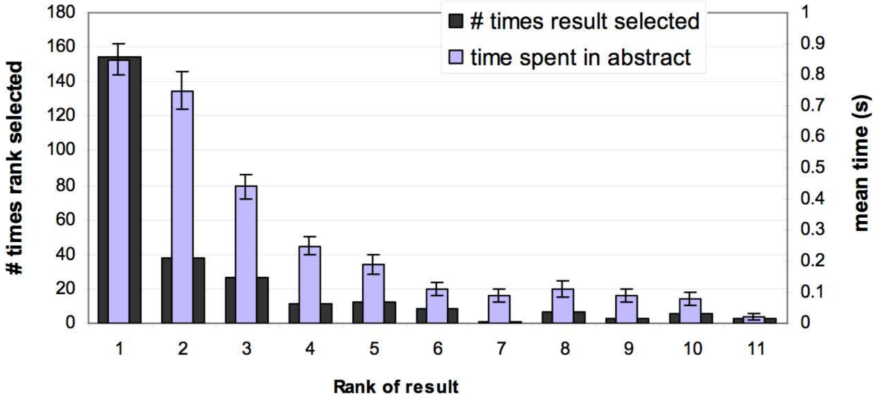
Overview

1. Recap
2. Intro vector space classification
3. Rocchio
4. kNN
5. Linear classifiers
6. $>$ two classes
7. Feature selection

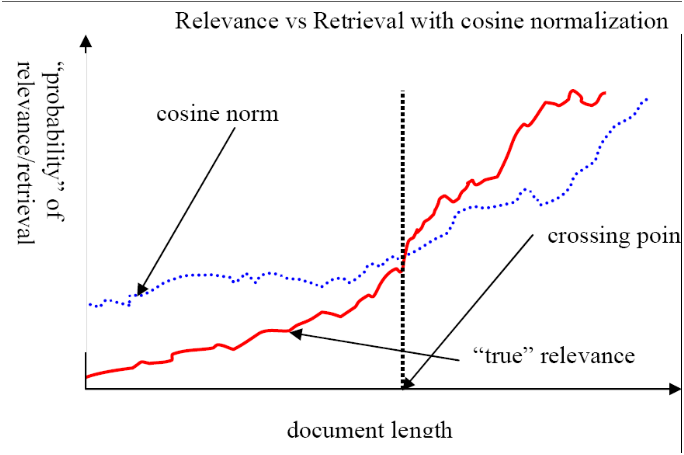
Outline

Looking vs. Clicking



- Users view results one and two more often / thoroughly
- Users click most frequently on result one

Pivot normalization



source: Lillian Lee

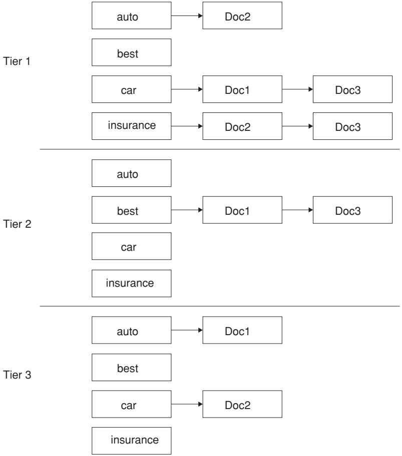
Selecting k top scoring documents in $O(N \log k)$

- Goal: Keep the k top documents seen so far
- Use a binary min heap
- To process a new document d' with score s':
  - Get current minimum h_m of heap (in $O(1)$)
  - If $s' \leq h_m$ skip to next document
  - If $s' > h_m$ heap-delete-root (in $O(\log k)$)
  - Heap-add d'/s' (in $O(1)$)
  - Reheapify (in $O(\log k)$)

Heuristics for finding the top k even faster

- Document-at-a-time processing
  - We complete computation of the query-document similarity score of document d_i before starting to compute the query-document similarity score of d_{i+1}.
  - Requires a consistent ordering of documents in the postings lists
- Term-at-a-time processing
  - We complete processing the postings list of query term t_i before starting to process the postings list of t_{i+1}.
  - Requires an accumulator for each document "still in the running"
- The most effective heuristics switch back and forth between term-at-a-time and document-at-a-time processing.

Tiered index



Take-away today

- Vector space classification: Basic idea of doing text classification for documents that are represented as vectors
- Rocchio classifier: Rocchio relevance feedback idea applied to text classification
- *k* nearest neighbor classification
- Linear classifiers
- More than two classes
- Feature selection for text classification: How to select a subset of available dimensions

Outline

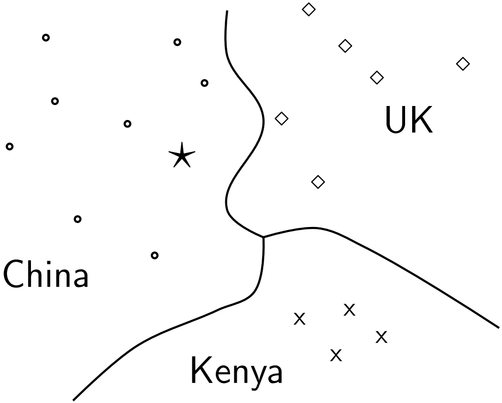
Recall vector space representation

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 100,000s of dimensions
- Normalize vectors (documents) to unit length
- How can we do classification in this space?

Vector space classification

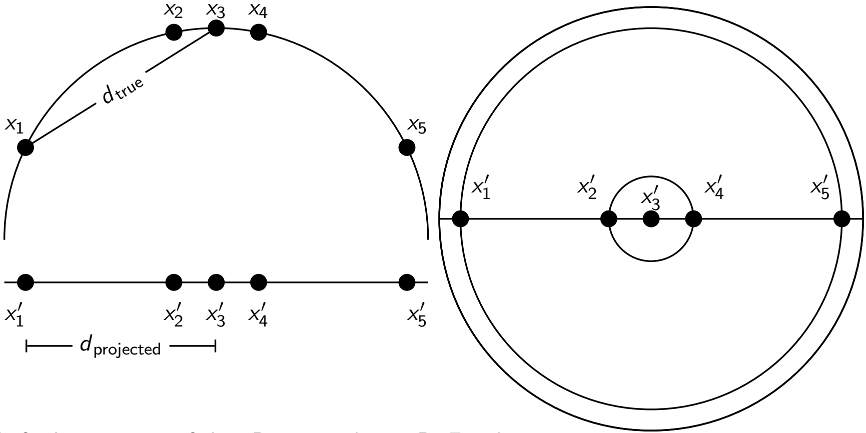
- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a contiguous region.
- Premise 2: Documents from different classes don't overlap.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space



Should the document $\star$ be assigned to *China*, *UK* or *Kenya*? Find separators between the classes Based on these separators: $\star$ should be assigned to *China* How do we find separators that do a good job at classifying new documents like $\star$? – Main topic of today

Aside: 2D/3D graphs can be misleading



Left: A projection of the 2D semicircle to 1D. For the points x_1, x_2, x_3, x_4, x_5 at x coordinates $-0.9, -0.2, 0, 0.2, 0.9$ the distance $|x_2 x_3| \approx 0.201$ only differs by 0.5% from $|x'_2 x'_3| = 0.2$; but $|x_1 x_3| / |x'_1 x'_3| = d_{\text{true}}/d_{\text{projected}} \approx 1.06/0.9 \approx 1.18$ is an example of a large distortion (18%) when projecting a large area. *Right:* The corresponding projection of the 3D hemisphere to 2D.

Outline

Relevance feedback

- In relevance feedback, the user marks documents as relevant/nonrelevant.
- Relevant/nonrelevant can be viewed as classes or categories.
- For each document, the user decides which of these two classes is correct.
- The IR system then uses these class assignments to build a better query ("model") of the information need ...
- ... and returns better documents.
- Relevance feedback is a form of text classification.

Using Rocchio for vector space classification

- The principal difference between relevance feedback and text classification:
  - The training set is given as part of the input in text classification.
  - It is interactively created in relevance feedback.

Rocchio classification: Basic idea

- Compute a centroid for each class
  - The centroid is the average of all documents in the class.
- Assign each test document to the class of its closest centroid.

Recall definition of centroid

$$\vec{\mu}(c) = \frac{1}{|D_c|} \sum_{d \in D_c} \vec{v}(d)$$

where D_c is the set of all documents that belong to class c and

$\vec{v}(d)$ is the vector space representation of d.

Rocchio algorithm

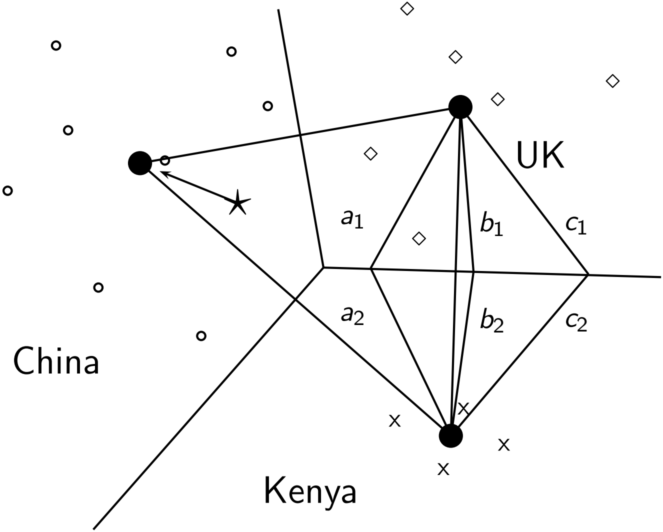
$\text{TrainRocchio}(\mathbb{C}, \mathbb{D})$

for each $c_j \in \mathbb{C}$
do $D_j \leftarrow \{d : \langle d, c_j \rangle \in \mathbb{D}\}$
$\quad \vec{\mu}_j \leftarrow \frac{1}{|D_j|} \sum_{d \in D_j} \vec{v}(d)$
return $\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}$

$\text{ApplyRocchio}(\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}, d)$

return $\arg\min_j |\vec{\mu}_j - \vec{v}(d)|$

Rocchio illustrated : $a_1 = a_2, b_1 = b_2, c_1 = c_2$



Rocchio properties

- Rocchio forms a simple representation for each class: the centroid
  - We can interpret the centroid as the prototype of the class.
- Classification is based on similarity to / distance from centroid/prototype.
- Does not guarantee that classifications are consistent with the training data!

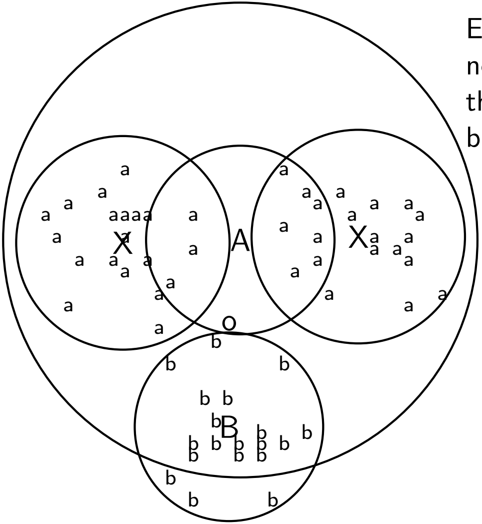
Time complexity of Rocchio

mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\text{ave}} + \lvert\mathbb{C}\rvert\lvert V\rvert) \approx \Theta(\lvert\mathbb{D}\rvert L_{\text{ave}})$
testing	$\Theta(L_{\text{a}} + \lvert\mathbb{C}\rvert M_{\text{a}}) \approx \Theta(\lvert\mathbb{C}\rvert M_{\text{a}})$

Rocchio vs. Naive Bayes

- In many cases, Rocchio performs worse than Naive Bayes.
- One reason: Rocchio does not handle nonconvex, multimodal classes correctly.

Rocchio cannot handle nonconvex, multimodal classes



Exercise: Why is Rocchio not expected to do well for the classification task a vs. b here?

- A is centroid of the a's, B is centroid of the b's.
- The point o is closer to A than to B.
- But o is a better fit for the b class.
- A is a multimodal class with two prototypes.
- But in Rocchio we only have one prototype.

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kNN classification

- kNN classification is another vector space classification method.
- It also is very simple and easy to implement.
- kNN is more accurate (in most cases) than Naive Bayes and Rocchio.
- If you need to get a pretty accurate classifier up and running in a short time . . .
- . . . and you don't care about efficiency that much . . .
- . . . use kNN.

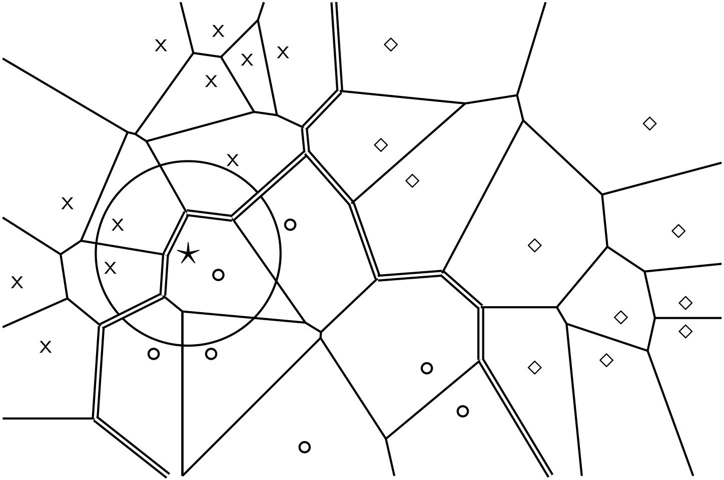
kNN classification

- kNN = k nearest neighbors
- kNN classification rule for $k = 1$ (1NN): Assign each test document to the class of its nearest neighbor in the training set.
- 1NN is not very robust – one document can be mislabeled or atypical.
- kNN classification rule for $k > 1$ (kNN): Assign each test document to the majority class of its k nearest neighbors in the training set.
- Rationale of kNN: contiguity hypothesis
  - We expect a test document d to have the same label as the training documents located in the local region surrounding d.

Probabilistic kNN

- Probabilistic version of kNN: $P(c|d) =$ fraction of k neighbors of d that are in c
- kNN classification rule for probabilistic kNN: Assign d to class c with highest $P(c|d)$

kNN is based on Voronoi tessellation

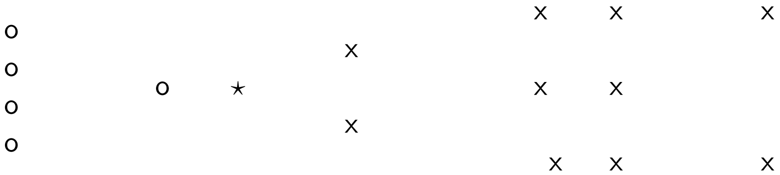


kNN algorithm

$\textsc{Train-kNN}(\mathbb{C}, \mathbb{D})$
$\mathbb{D}' \leftarrow \textsc{Preprocess}(\mathbb{D})$
$k \leftarrow \textsc{Select-k}(\mathbb{C}, \mathbb{D}')$
return $\mathbb{D}', k$

$\textsc{Apply-kNN}(\mathbb{D}', k, d)$
$S_k \leftarrow \textsc{ComputeNearestNeighbors}(\mathbb{D}', k, d)$
for each $c_j \in \mathbb{C}(\mathbb{D}')$
do $p_j \leftarrow |S_k \cap c_j|/k$
return $\arg\max_j p_j$

Exercise



How is star classified by:

(i) 1-NN (ii) 3-NN (iii) 9-NN (iv) 15-NN (v) Rocchio?

Time complexity of kNN

kNN with preprocessing of training set

training $\Theta(|\mathbb{D}|L_{\mathsf{ave}})$

testing $\Theta(L_{\mathsf{a}} + |\mathbb{D}|M_{\mathsf{ave}}M_{\mathsf{a}}) = \Theta(|\mathbb{D}|M_{\mathsf{ave}}M_{\mathsf{a}})$

- kNN test time proportional to the size of the training set!
- The larger the training set, the longer it takes to classify a test document.
- kNN is inefficient for very large training sets.

kNN: Discussion

- No training necessary
  - But linear preprocessing of documents is as expensive as training Naive Bayes.
  - We always preprocess the training set, so in reality training time of kNN is linear.
- kNN is very accurate if training set is large.
- Optimality result: asymptotically zero error if Bayes rate is zero.
- But kNN can be very inaccurate if training set is small.

Outline

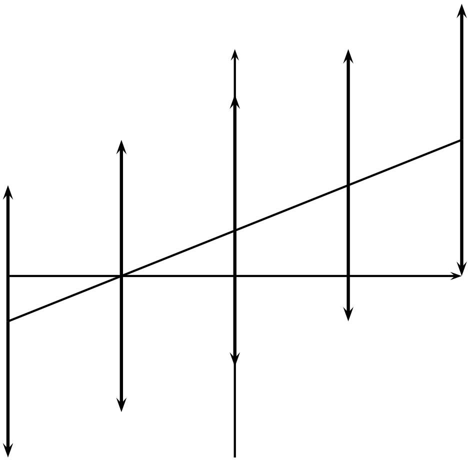
Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum $\sum_i w_i x_i$ of the feature values.
  - Classification decision: $\sum_i w_i x_i > \theta$?
  - ...where θ (the threshold) is a parameter.
- (First, we only consider binary classifiers.)
- Geometrically, this corresponds to a line (2D), a plane (3D) or a hyperplane (higher dimensionalities), the separator.
- We find this separator based on training set.
- Methods for finding separator: Perceptron, Rocchio, Naive Bayes – as we will explain on the next slides
- Assumption: The classes are linearly separable.

A linear classifier in 1D

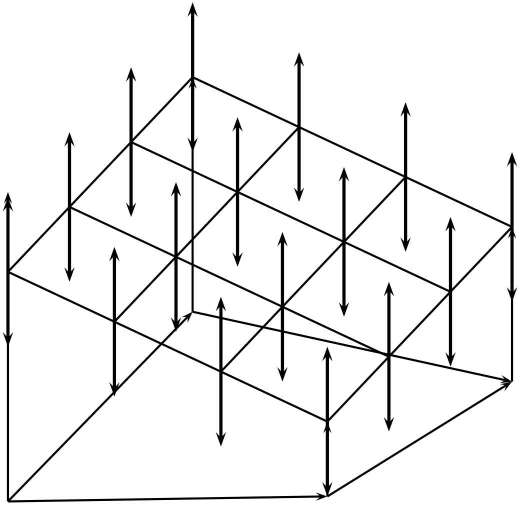
- A linear classifier in 1D is a point described by the equation $w_1 d_1 = \theta$
- The point at θ / w_1
- Points (d_1) with $w_1 d_1 \geq \theta$ are in the class c.
- Points (d_1) with $w_1 d_1 < \theta$ are in the complement class $\overline{c}$.

A linear classifier in 2D



- A linear classifier in 2D is a line described by the equation $w_1 d_1 + w_2 d_2 = \theta$
- Example for a 2D linear classifier
- Points $(d_1 \ d_2)$ with $w_1 d_1 + w_2 d_2 \geq \theta$ are in the class c.
- Points $(d_1 \ d_2)$ with $w_1 d_1 + w_2 d_2 < \theta$ are in the complement class $\overline{c}$.

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation $w_1 d_1 + w_2 d_2 + w_3 d_3 = \theta$
- Example for a 3D linear classifier
- Points $(d_1\ d_2\ d_3)$ with $w_1 d_1 + w_2 d_2 + w_3 d_3 \geq \theta$ are in the class c.
- Points $(d_1\ d_2\ d_3)$ with $w_1 d_1 + w_2 d_2 + w_3 d_3 < \theta$ are in the complement class $\overline{c}$.

Rocchio as a linear classifier

- Rocchio is a linear classifier defined by:

$$\sum_{i=1}^{M} w_i d_i = \vec{w}\vec{d} = \theta$$

where $\vec{w}$ is the normal vector $\vec{\mu}(c_1) - \vec{\mu}(c_2)$ and $\theta = 0.5 * (|\vec{\mu}(c_1)|^2 - |\vec{\mu}(c_2)|^2)$.

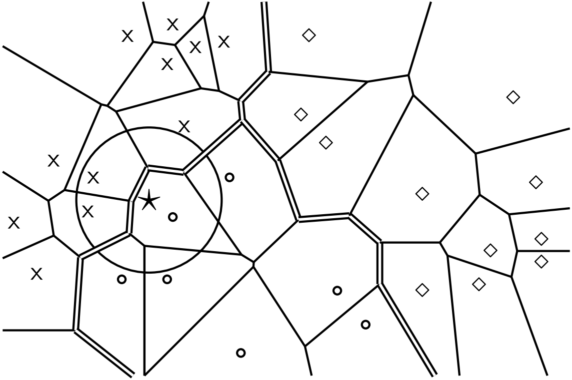
Naive Bayes as a linear classifier

Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^{M} w_i d_i = \theta$$

where $w_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$, $d_i =$ number of occurrences of t_i in d, and $\theta = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$. Here, the index i, $1 \leq i \leq M$, refers to terms of the vocabulary (not to positions in d as k did in our original definition of Naive Bayes)

kNN is not a linear classifier



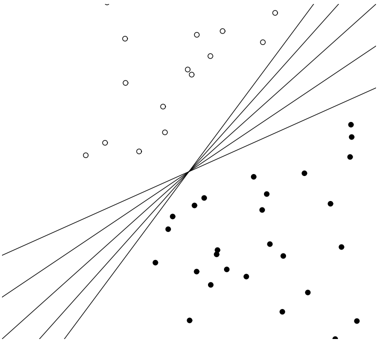
- Classification decision based on majority of k nearest neighbors.
- The decision boundaries between classes are piecewise linear ...
- ...but they are in general not linear classifiers that can be described as $\sum_{i=1}^{M} w_i d_i = \theta$.

Example of a linear two-class classifier

t_i	w_i	d_{1i}	d_{2i}	t_i	w_i	d_{1i}	d_{2i}
prime	0.70	0	1	dlrs	-0.71	1	1
rate	0.67	1	0	world	-0.35	1	0
interest	0.63	0	0	sees	-0.33	0	0
rates	0.60	0	0	year	-0.25	0	0
discount	0.46	1	0	group	-0.24	0	0
bundesbank	0.43	0	0	dlr	-0.24	0	0

- This is for the class *interest* in Reuters-21578.
- For simplicity: assume a simple 0/1 vector representation
- d_1: "rate discount dlrs world"
- d_2: "prime dlrs"
- $\theta = 0$
- Exercise: Which class is d_1 assigned to? Which class is d_2 assigned to?
- We assign document $\vec{d}_1$ "rate discount dlrs world" to *interest* since $\vec{w}^T \vec{d}_1 = 0.67 \cdot 1 + 0.46 \cdot 1 + (-0.71) \cdot 1 + (-0.35) \cdot 1 = 0.07 > 0 = \theta$.
- We assign $\vec{d}_2$ "prime dlrs" to the complement class (not in *interest*) since $\vec{w}^T \vec{d}_2 = -0.01 \leq \theta$.

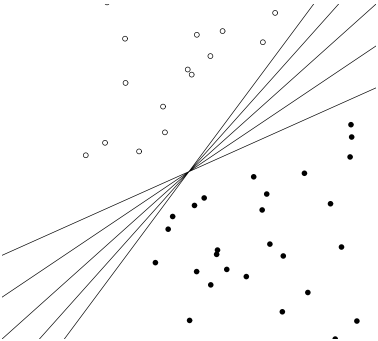
Which hyperplane?



Learning algorithms for vector space classification

- In terms of actual computation, there are two types of learning algorithms.
- (i) Simple learning algorithms that estimate the parameters of the classifier directly from the training data, often in one linear pass.
  - Naive Bayes, Rocchio, kNN are all examples of this.
- (ii) Iterative algorithms
  - Support vector machines
  - Perceptron (example available as PDF on website: http://ifnlp.org/ir/pdf/p.pdf)
- The best performing learning algorithms usually require iterative learning.

Which hyperplane?



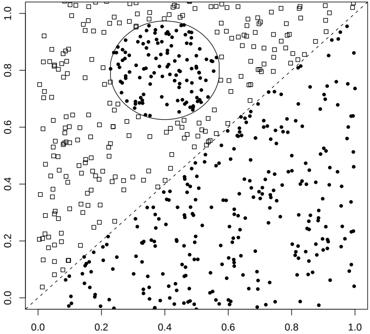
Which hyperplane?

- For linearly separable training sets: there are infinitely many separating hyperplanes.
- They all separate the training set perfectly ...
- ...but they behave differently on test data.
- Error rates on new data are low for some, high for others.
- How do we find a low-error separator?
- Perceptron: generally bad; Naive Bayes, Rocchio: ok; linear SVM: good

Linear classifiers: Discussion

- Many common text classifiers are linear classifiers: Naive Bayes, Rocchio, logistic regression, linear support vector machines etc.
- Each method has a different way of selecting the separating hyperplane
  - Huge differences in performance on test documents
- Can we get better performance with more powerful nonlinear classifiers?
- Not in general: A given amount of training data may suffice for estimating a linear boundary, but not for estimating a more complex nonlinear boundary.

A nonlinear problem



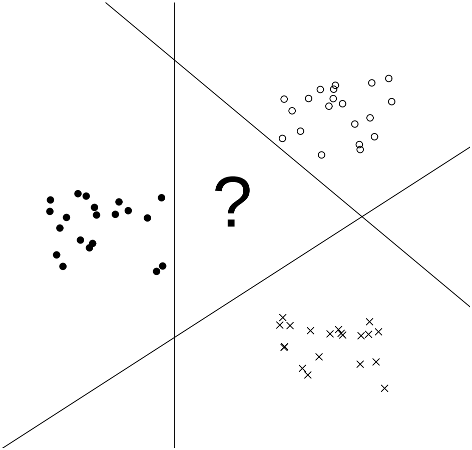
- Linear classifier like Rocchio does badly on this task.
- kNN will do well (assuming enough training data)

Which classifier do I use for a given TC problem?

- Is there a learning method that is optimal for all text classification problems?
- No, because there is a tradeoff between bias and variance.
- Factors to take into account:
  - How much training data is available?
  - How simple/complex is the problem? (linear vs. nonlinear decision boundary)
  - How noisy is the problem?
  - How stable is the problem over time?
    - For an unstable problem, it's better to use a simple and robust classifier.

Outline

How to combine hyperplanes for > 2 classes?



One-of problems

- One-of or multiclass classification
  - Classes are mutually exclusive.
  - Each document belongs to exactly one class.
  - Example: language of a document (assumption: no document contains multiple languages)

One-of classification with linear classifiers

- Combine two-class linear classifiers as follows for one-of classification:
  - Run each classifier separately
  - Rank classifiers (e.g., according to score)
  - Pick the class with the highest score

Any-of problems

- Any-of or multilabel classification
  - A document can be a member of 0, 1, or many classes.
  - A decision on one class leaves decisions open on all other classes.
  - A type of “independence” (but not statistical independence)
  - Example: topic classification
  - Usually: make decisions on the region, on the subject area, on the industry and so on “independently”

Any-of classification with linear classifiers

- Combine two-class linear classifiers as follows for any-of classification:
  - Simply run each two-class classifier separately on the test document and assign document accordingly

Outline

Feature selection

- In text classification, we usually represent documents in a high-dimensional space, with each dimension corresponding to a term.
- In this lecture: axis = dimension = word = term = feature
- Many dimensions correspond to rare words.
- Rare words can mislead the classifier.
- Rare misleading features are called noise features.
- Eliminating noise features from the representation increases efficiency and effectiveness of text classification.
- Eliminating features is called feature selection.

Example for a noise feature

- Let's say we're doing text classification for the class *China*.
- Suppose a rare term, say ARACHNOCENTRIC, has no information about *China* ...
- ...but all instances of ARACHNOCENTRIC happen to occur in *China* documents in our training set.
- Then we may learn a classifier that incorrectly interprets ARACHNOCENTRIC as evidence for the class *China*.
- Such an incorrect generalization from an accidental property of the training set is called overfitting.
- Feature selection reduces overfitting and improves the accuracy of the classifier.

Basic feature selection algorithm

$\textsc{SelectFeatures}(\mathbb{D}, c, k)$

$V \leftarrow \textsc{ExtractVocabulary}(\mathbb{D})$

$L \leftarrow []$

for each $t \in V$

do $A(t, c) \leftarrow \textsc{ComputeFeatureUtility}(\mathbb{D}, t, c)$

$\quad \textsc{Append}(L, \langle A(t, c), t\rangle)$

return $\textsc{FeaturesWithLargestValues}(L, k)$

How do we compute A, the feature utility?

Different feature selection methods

- A feature selection method is mainly defined by the feature utility measure it employs
- Feature utility measures:
  - Frequency – select the most frequent terms
  - Mutual information – select the terms with the highest mutual information
  - Mutual information is also called information gain in this context.
  - Chi-square (see book)

Mutual information

- Compute the feature utility $A(t, c)$ as the expected mutual information (MI) of term t and class c.
- MI tells us "how much information" the term contains about the class and vice versa.
- For example, if a term's occurrence is independent of the class (same proportion of docs within/without class contain the term), then MI is 0.
- Definition:

$$I(U;C) = \sum_{e_t \in \{1,0\}} \sum_{e_c \in \{1,0\}} P(U=e_t, C=e_c) \log_2 \frac{P(U=e_t, C=e_c)}{P(U=e_t)P(C=e_c)}$$

How to compute MI values

- Based on maximum likelihood estimates, the formula we actually use is:

$$
\begin{aligned}
I(U;C) &= \frac{N_{11}}{N}\log_2\frac{NN_{11}}{N_{1.}N_{.1}} + \frac{N_{01}}{N}\log_2\frac{NN_{01}}{N_{0.}N_{.1}} \\
&\quad + \frac{N_{10}}{N}\log_2\frac{NN_{10}}{N_{1.}N_{.0}} + \frac{N_{00}}{N}\log_2\frac{NN_{00}}{N_{0.}N_{.0}}
\end{aligned}
$$

- N_{10}: number of documents that contain t ($e_t = 1$) and are not in c ($e_c = 0$); N_{11}: number of documents that contain t ($e_t = 1$) and are in c ($e_c = 1$); N_{01}: number of documents that do not contain t ($e_t = 1$) and are in c ($e_c = 1$); N_{00}: number of documents that do not contain t ($e_t = 1$) and are not in c ($e_c = 1$); $N = N_{00} + N_{01} + N_{10} + N_{11}$.

MI example for *poultry*/EXPORT in Reuters

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\text{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\text{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Plug these values into formula:

$$
\begin{aligned}
I(U;C) &= \frac{49}{801{,}948}\log_2\frac{801{,}948\cdot 49}{(49+27{,}652)(49+141)} \\
&\quad+\frac{141}{801{,}948}\log_2\frac{801{,}948\cdot 141}{(141+774{,}106)(49+141)} \\
&\quad+\frac{27{,}652}{801{,}948}\log_2\frac{801{,}948\cdot 27{,}652}{(49+27{,}652)(27{,}652+774{,}106)} \\
&\quad+\frac{774{,}106}{801{,}948}\log_2\frac{801{,}948\cdot 774{,}106}{(141+774{,}106)(27{,}652+774{,}106)} \\
&\approx 0.000105
\end{aligned}
$$

MI feature selection on Reuters

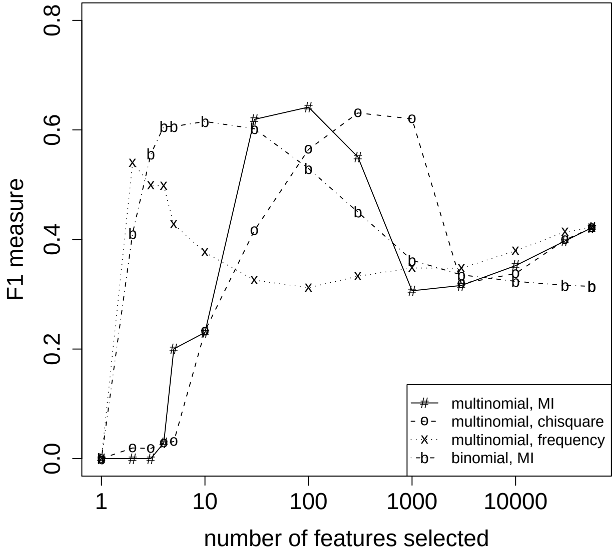
Class: *coffee*

term	MI
COFFEE	0.0111
BAGS	0.0042
GROWERS	0.0025
KG	0.0019
COLOMBIA	0.0018
BRAZIL	0.0016
EXPORT	0.0014
EXPORTERS	0.0013
EXPORTS	0.0013
CROP	0.0012

Class: *sports*

term	MI
SOCCER	0.0681
CUP	0.0515
MATCH	0.0441
MATCHES	0.0408
PLAYED	0.0388
LEAGUE	0.0386
BEAT	0.0301
GAME	0.0299
GAMES	0.0284
TEAM	0.0264

Naive Bayes: Effect of feature selection



(multinomial = multinomial Naive Bayes, binomial = Bernoulli Naive Bayes)

Feature selection for Naive Bayes

- In general, feature selection is necessary for Naive Bayes to get decent performance.
- Also true for most other learning methods in text classification: you need feature selection for optimal performance.

Exercise

(i) Compute the "export"/POULTRY contingency table for the "Kyoto"/JAPAN in the collection given below. (ii) Make up a contingency table for which MI is 0 – that is, term and class are independent of each other. "export"/POULTRY table:

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\text{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\text{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Collection:

	docID	words in document	in $c =$ Japan?
training set	1	Kyoto Osaka Taiwan	yes
	2	Japan Kyoto	yes
	3	Taipei Taiwan	no
	4	Macao Taiwan Shanghai	no
	5	London	no

Take-away today

- Vector space classification: Basic idea of doing text classification for documents that are represented as vectors
- Rocchio classifier: Rocchio relevance feedback idea applied to text classification
- k nearest neighbor classification
- Linear classifiers
- More than two classes
- Feature selection for text classification: How to select a subset of available dimensions

Resources

- Chapter 13 of IIR (feature selection)
- Chapter 14 of IIR
- Resources at `http://ifnlp.org/ir`
  - Perceptron example
  - General overview of text classification: Sebastiani (2002)
  - Text classification chapter on decision tress and perceptrons: Manning & Schütze (1999)
  - One of the best machine learning textbooks: Hastie, Tibshirani & Friedman (2003)